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Analyzing inter-modal competition between high speed rail and conventional transport systems: A game theoretic approach

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Abstract

A methodology is developed in order to assess the viability of transport infrastructure investment in the form of High Speed Rail (HSR). Public transportation mode operators such as HSR, conventional trains and buses, maximize their profits by varying prices and frequency for a given demand and infrastructure cost. In this study, the price competition between different operators is taken into consideration and the change in the existing market equilibrium due to the entry of the new mode is studied using the game theoretic approach. Hypothetical data for a particular route is used for game-based analysis. In this multiplayer game, the effect of introducing the new mode of transport on the Nash equilibrium is studied taking into account the competition between the other modes of transportation. The analysis of market share for the modes has been carried out using heterogeneity of the passengers based on the concept of Value-of-Time (VOT). The passengers are assumed to be intelligent and rational in choosing the mode that minimizes their generalized travel cost, which is a function of travel time weighted by the individual VOT and the monetary cost associated with the mode of travel. Thus, different combinations of entry and response strategies are studied for HSR and existing modes, and the impact of introduction of HSR is assessed in terms of profit, thus, reflecting on the sustainability and financial viability of the transport infrastructure investment.

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Keywords: Intermodal competition; Game theory; High Speed Rail (HSR); Value-of-Time(VOT).

1. HSR in Indian Context

In India, rapid urbanization, growing economy and increasing per capita income has caused increasing growth in intercity travel. High speed rail (HSR) system has been globally proven to be an efficient transportation mode

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to fulfill the demand gap for faster intercity movement of passenger traffic. Thus, in order to cater to the ever increasing passenger traffic and demand for better services, the Government of India is exploring the option of introducing HSR system as a sustainable mode of transportation. India has one of the largest rail networks in the world. Currently, India does not have any high-speed rail lines capable of supporting speeds of 200 km/h (124 mph) or more. However, high-speed corridors have been proposed and are under prefeasibility studies. Six corridors have already been identified for technical studies on setting up of HSR as shown in Table 1.

Table 1. Proposed corridors for technical studies (Source: Vision 2020, Indian Railways)

High-Speed Corridor	Route	Stations	Speed (kmph)	Length (km)
Howrah-Haldia	Howrah-Haldia	TBD	250-300	135
Delhi-Patna	Delhi-Agra-Kanpur-Lucknow-Varanasi-Patna	TBD	200-350	991
Delhi-Amritsar	Delhi-Chandigarh-Amritsar	TBD	TBD	450
Hyderabad-Chennai	Hyderabad-Dornakal-Vijayawada-Chennai	TBD	TBD	664
Pune-Mumbai-Ahmedabad	Pune-Mumbai-Ahmedabad	7	300-350	650
Chennai-Bangalore-Coimbatore-Ernakulam	Chennai-Bangalore-Coimbatore-Ernakulam	TBD	350	649

2. Literature Review

Wang and Yang (2005) analyze the competition between HSR and conventional rail system in Taiwan. The concept of linear city is used for computing demand and model passenger's behaviour. They have developed a game theoretic model to infer about the rail system's pricing strategy under a two-part tariff pricing strategy by solving for the ticket prices under Nash equilibrium, the objective function being the generalized travel cost function of the passengers. Using a real data based example, they demonstrated that when HSR's operating cost increases, both the modes increase their fare. They have predicted that a rise in passenger's time value increases the demand for HSR but decreases the demand for conventional rail system. Computational results showed that HSR raises its fare to remain in profit whereas the conventional rail system keeps its fares low to maintain the mode share. Concept of linear city cannot be adopted in every scenario and competition with other modes is not taken into account.

The effect of value of time (VOT) distribution on price and quality competition among differentiated bus services in a simple corridor is investigated by Yang, Yan Kong and Meng (2001). They assume that for a given continuous distribution of VOT, users will minimize their individual generalized trip cost and divide themselves in the available modes of transportation which differ in monetary cost and travel time. It was concluded that VOT distribution significantly affect the profitability of various differentiated bus services. Results showed that as the VOT standard deviation becomes smaller i.e. more homogeneous and users are more confined to higher VOT, normal bus services will suffer a deficit in business and will be driven out of the market. Thus assumption of VOT distribution parameters is crucial and will vary from corridor to corridor.

Yang, Yan Kong and Meng (2001); Zhang, Su, Peng and Yao (2010) assess the impact of deregulation of the bus market in terms of profits and social cost to the society by modelling the strategic interactions between the operators in the bus market. A hypothetical case is constructed to test the model where the competition is investigated for two scenarios, single period formulation and multi period formulation. Zhang, Su, Peng and Yao, (2010) calculate demand equilibrium using the VOT concept where the VOT distribution is assumed to be uniform. They assume that the incumbent moves first choosing its fare and frequency by keeping in mind three strategies i.e. accommodation, deterrence and monopolist. The entrant has two options of entering the market or staying out by deciding its fare and frequency which maximizes its profit after the incumbent has moved. They concluded that deterrence is the dominant strategy in many market scenarios which is best for the society whereas

accommodation occurs when the sunk cost is low and demand is high. Natural monopoly arises when sunk cost is high and route density is low.

Shyr and Hung (2010) state that if Taiwan High speed rail continues to increase its service frequency and lower its promotion prices, the chances of airline market to compete with HSR is very slim. They predict that to maintain profitability, airlines would have to unify as an alliance and cut their daily flights by 50%. Mode shares are estimated using discrete choice modelling by conducting revealed preference and stated preference survey. The solution to Nash equilibrium is derived by finding the upper bound fare which maximizes the social welfare. They incorporate co-operative game theory to study the coalition structures between the airlines. For given payoff values, Shapley value is computed in order to solve the profit distribution problem.

3. The Model

A single origin-destination pair is considered with HSR as an entrant into the market having existent players: conventional trains and buses. Additional modes like domestic airline services can be added to the model developed and hence it is not considered initially without loss of generality. The bus transport mode is considered as the incumbent and HSR upon its arrival will target the passengers having higher VOT causing a redistribution of market share. Thus, in the current study, conventional train mode is ignored in the competition as it targets only passengers having low VOT.

The modeling aspect of the study consists of two equilibrium points: Demand Equilibrium and Market Equilibrium, hence the model is a two-level equilibrium model. The first level is the demand equilibrium among the users, assuming that they are heterogeneous in terms of Value-of-time (VOT). Yang, Yan Kong and Meng (2001) modeled the demand equilibrium in competitive bus services defined as when no user can reduce individual trip cost by unilaterally switching his travel mode. The demand equilibrium was obtained by partitioning the distribution of VOT into market segments corresponding to different quality of services. The same approach is adopted to model the mode choice behaviour in this study.

The second level determines the equilibrium strategies of the incumbent and entrant through a two-player two-stage extensive form game. The incumbent's strategy is either (a) Accommodation -Maximize profit by revising its fare and frequency as a best response to the given fare and frequency of the entrant; or (b) Monopolist – Setting fare and frequency for profit maximization as if there is no potential entrant. The entrant's strategy is whether to (a) enter; or (b) stay out. If it decides to enter, its fare and frequency will be a best response to the given fare and frequency of the incumbent.

3.1. Demand Equilibrium

Individual user's modal choices for a given set of strategies, in terms of fare and frequency offered by the operator(s), is assumed to be based on the generalized cost of the journey which is a function of the monetary cost and travel time weighted by the VOT.

Let

$\{f_t, F_t\}$ be the fare and frequency of the train;

$\{f_b, F_b\}$ be the fare and frequency of the bus;

$\{f_h, F_h\}$ be the fare and frequency of the entrant (HSR);

T_t, T_b and T_h be the travel times of train, bus and HSR, respectively which include in-vehicle travel time and access time; and

$G_t(\tau)$, $G_b(\tau)$ and $G_h(\tau)$ be the generalised costs for a user with VOT value of τ to travel in train, bus and HSR respectively.

$$G_t(\tau) = f_t + (T_t \times \tau) \quad (1)$$

$$G_b(\tau) = f_b + (T_b \times \tau) \quad (2)$$

$$G_h(\tau) = f_h + (T_h \times \tau) \quad (3)$$

Let $f(\tau)$ be the probability density function of the distribution (assumed uniform for simplicity) of VOTs across the user population, i.e. $\tau \in [\theta_1; \theta_2]$

$$f(\tau) = \frac{1}{\theta_2 - \theta_1} \text{ where } \theta_1 \leq \tau \leq \theta_2 \quad (4)$$

3.1.1 Demand equilibrium without market entry

The following relationships between the fares and travel times on the two existing modes are assumed:

$$f_b > f_t \quad (5)$$

$$T_b < T_t \quad (6)$$

Without the entry of HSR, the market can be partitioned into two segments - train users and bus users. The cut-off VOT is at

$$G_t(\bar{\tau}_{tb}) = G_b(\bar{\tau}_{tb}) \quad (7)$$

$$f_t + (T_t \times \bar{\tau}_{tb}) = f_b + (T_b \times \bar{\tau}_{tb}) \quad (8)$$

$$\bar{\tau}_{tb} = \frac{(f_b - f_t)}{(T_t - T_b)} \quad (9)$$

The number of users travelling on bus and train are:

$$Q_b = Q \int_{\bar{\tau}_{tb}}^{\theta_2} f(x) dx \quad (10)$$

$$Q_t = Q \int_{\theta_1}^{\bar{\tau}_{tb}} f(x) dx \quad (11)$$

3.1.2 Demand equilibrium with market entry

If the entrant decides to enter, the following relationships between the fares and travel times on the three transport modes are assumed:

$$f_h > f_b > f_t \quad (12)$$

$$T_h < T_b < T_t \quad (13)$$

Users with VOT such that $G_t(\tau) < G_b(\tau)$ and $G_t(\tau) < G_h(\tau)$ will strictly prefer the existing train. The upper cut-off of VOT for these travellers ($\bar{\tau}_{tb}$) is at

$$G_t(\bar{\tau}_{tb}) = G_b(\bar{\tau}_{tb}) \quad (14)$$

$$f_t + (T_t \times \bar{\tau}_{tb}) = f_b + (T_b \times \bar{\tau}_{tb}) \quad (15)$$

$$\bar{\tau}_{tb} = \frac{(f_b - f_t)}{(T_t - T_b)} \quad (16)$$

Users with VOT such that $G_h(\tau) < G_t(\tau)$ and $G_h(\tau) < G_b(\tau)$ will strictly prefer HSR. The lower cut-off of VOT for these travellers ($\bar{\tau}_{bh}$) is at

$$G_h(\bar{\tau}_{bh}) = G_b(\bar{\tau}_{bh}) \quad (17)$$

$$f_h + (T_h \times \bar{\tau}_{bh}) = f_b + (T_b \times \bar{\tau}_{bh}) \quad (18)$$

$$\bar{\tau}_{bh} = \frac{(f_h - f_b)}{(T_b - T_h)} \quad (19)$$

The above cut-off VOT values in effect partition the market into three segments. Under the assumed relationships between fares and travel times, as described by (12) and (13), HSR becomes the most up-market mode among the three transport modes which will be taken by users with the highest VOT in the range. The bus service will be taken by users with VOT in the middle range who find HSR costly and consider the bus service affordable. Finally, users with the lowest VOT will consider both HSR and the bus service costly thereby choosing the train as their preferred mode of transport. At demand equilibrium, the market shares of each of the three modes for a given total number of Q users are as follows:

$$Q_t = Q \int_{\theta_1}^{\bar{\tau}_{tb}} f(x) dx \quad (20)$$

$$Q_b = Q \int_{\bar{\tau}_{tb}}^{\bar{\tau}_{bh}} f(x) dx \quad (21)$$

$$Q_h = Q \int_{\bar{\tau}_{bh}}^{\theta_2} f(x) dx \quad (22)$$

4. Market Equilibrium

The problem of the optimum fare and frequency of the bus and HSR is represented as an extensive form game with complete information. The game is formulated in two stages. The first stage is where bus makes a move by setting its fare and frequency to maximize its profit in the market. In the second stage, HSR observes the strategy of bus and decides its own strategy, whether to enter in the market or stay out. If it decides to enter, it calculates its fare and frequency that maximizes its profit in presence of bus.

The two decision variables fare and frequency of bus and HSR respectively define a continuous two-dimensional space, thus, theoretically providing both players an infinite number of possible actions so that it would be impossible to represent the game in extensive form. However, we can simplify the game by first classifying all the possible strategies of the bus into two strategy sets, namely, accommodation and monopolist. In monopolist strategy, bus sets the fare and frequency for profit maximization as if there is no potential entrant while in accommodation strategy, profit obtained by the bus is maximized given that the HSR is going to enter and is best responding to the given fare and frequency of the bus.

For each strategy adopted by bus, HSR finds the optimum fare and frequency that maximizes its own profit. If the optimum value for profit is positive then HSR decides to enter the market, else it stays out of it. Now that the game is defined, to attain the equilibrium strategies, we need to look at the profit functions. Profit is defined as the difference between total revenue generated and total cost incurred. Here, total revenue is obtained as product of fare and market share where market share can be viewed as number of people travelling through the mode and cost includes fixed and variable costs. For HSR an additional cost of initial investment for developing infrastructure, called sunk cost, is involved.

We now define the profit maximization functions in detail. In the profit payoffs corresponding to different strategies, for instance, the subscript 'b' represent the mode, the superscript 'mon' represents the strategy of bus, and the subscript of the superscript 'in' represents the strategy of HSR.

In the monopolist strategy, the maximization problem for bus is:

Maximize

$$\pi_b^{monout} = \pi_b(f_b; F_b) = Q_{b1}(f_b; F_b)f_b - C_{vb}F_b - C_{fb} \quad (23)$$

such that

$$Q_{b1}(f_b; F_b) \leq F_b h_b \quad (24)$$

where the constraint is the capacity restriction on the bus.

From this, we obtain the fare and frequency of bus given the current market conditions without HSR. Based on this fare and frequency, HSR maximizes its profit through the optimization problem:

Maximize

$$\pi_H(f_h; F_h) = Q_{h2}(f_h; F_h; f_b; F_b)f_h - C_{vh}F_h - C_{fh} - C_{sh} \quad (25)$$

such that

$$Q_{h2}(f_h; F_h; f_b; F_b) \leq F_h H_h \quad (26)$$

HSR enters if $\pi_h > 0$ ($\pi_h^{monin} = \pi_h$), otherwise stays out ($\pi_h^{monout} = 0$). Now if HSR enters, a part of the market share of bus is captured by HSR, and therefore the profit of bus will change though the fare and frequency do not change as it is acting as a monopolist. The profit of bus after the entry of HSR is given by:

$$\pi_b^{monin} = \pi_b(f_b; F_b) = Q_{b2}(f_b; F_b)f_b - C_{vb}F_b - C_{fb} \quad (27)$$

If HSR stays out, the market share is not affected and the profit of bus remains the same as obtained from solution of (23).

In the accommodation strategy, before entry of HSR, bus will set its profit maximizing price and frequency considering the best response of HSR and the equilibrium price and frequency for the accommodation strategy is obtained by iterative solution to the optimization problems:

Maximize

$$\pi_h(f_h; F_h | f_b; F_b) = Q_{h2}(f_h; F_h; f_b; F_b)f_h - C_{vh}F_h - C_{fh} - C_{sh} \quad (28)$$

such that

$$Q_{h2}(f_h; F_h; f_b; F_b) \leq F_h H_h \quad (29)$$

Maximize

$$\pi_b(f_b; F_b | f_h; F_h) = Q_{b2}(f_b; F_b; f_h; F_h)f_b - C_{vb}F_b - C_{fb} \quad (30)$$

such that

$$Q_{b2}(f_b; F_b; f_h; F_h) \leq F_b H_b \quad (31)$$

HSR enters if $\pi_h > 0$ ($\pi_h^{acmin} = \pi_h$), otherwise stays out ($\pi_h^{acmout} = 0$). If HSR enters, the incumbents profit will be:

$$\pi_b^{acmin} = \pi_b(f_b; F_b | f_h; F_h) = Q_{b2}(f_b; F_b; f_h; F_h)f_b - C_{vb}F_b - C_{fb} \quad (32)$$

If HSR stays out the incumbents profit will be

$$\pi_b^{acmout} = \pi_b(f_b; F_b) = Q_{b1}(f_b; F_b)f_b - C_{vb}F_b - C_{fb} \quad (33)$$

We will explore the conditions for all the possible equilibriums in this game. The sub game perfect equilibrium solution can be determined by carrying out backward induction on the extensive form game. There are two possible sub games for HSR - one when bus follows accommodation strategy and another when it follows monopolist strategy. Considering the sub game corresponding to the accommodation strategy of bus, HSR selects the strategy that maximizes its profit, i.e. $\max(\pi_h^{acmin}, 0)$ and if it obtains positive profit the resulting payoffs passed to the root of the sub game as (π_h^{acmin}) . Similarly considering the monopolist strategy of bus, HSR again will choose the strategy that maximizes its profit, i.e. $\max(\pi_h^{monin}; 0)$. If the profit is positive given the

parameters of the game, the strategy selected is to enter and the payoffs passed to the root of the sub game is (π_h^{monin}) . Moving higher in the tree, now we need to find the equilibrium for the sub game corresponding to bus. With payoffs π_b^{acmin} and π_b^{monin} for the accommodation and monopolist strategies, bus will adopt the strategy which will give maximum profit, i.e. $\max(\pi_b^{acmin} ; \pi_b^{monin})$. This gives us a sub game perfect Nash Equilibrium for the game represented by the tree.

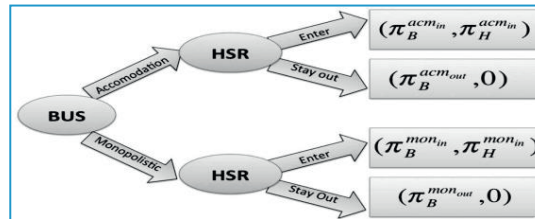


Figure 1: Extensive form game representation

5. Numerical Results and interpretations

The market equilibrium is calculated in the form of profits obtained for the optimal fare and frequency for both

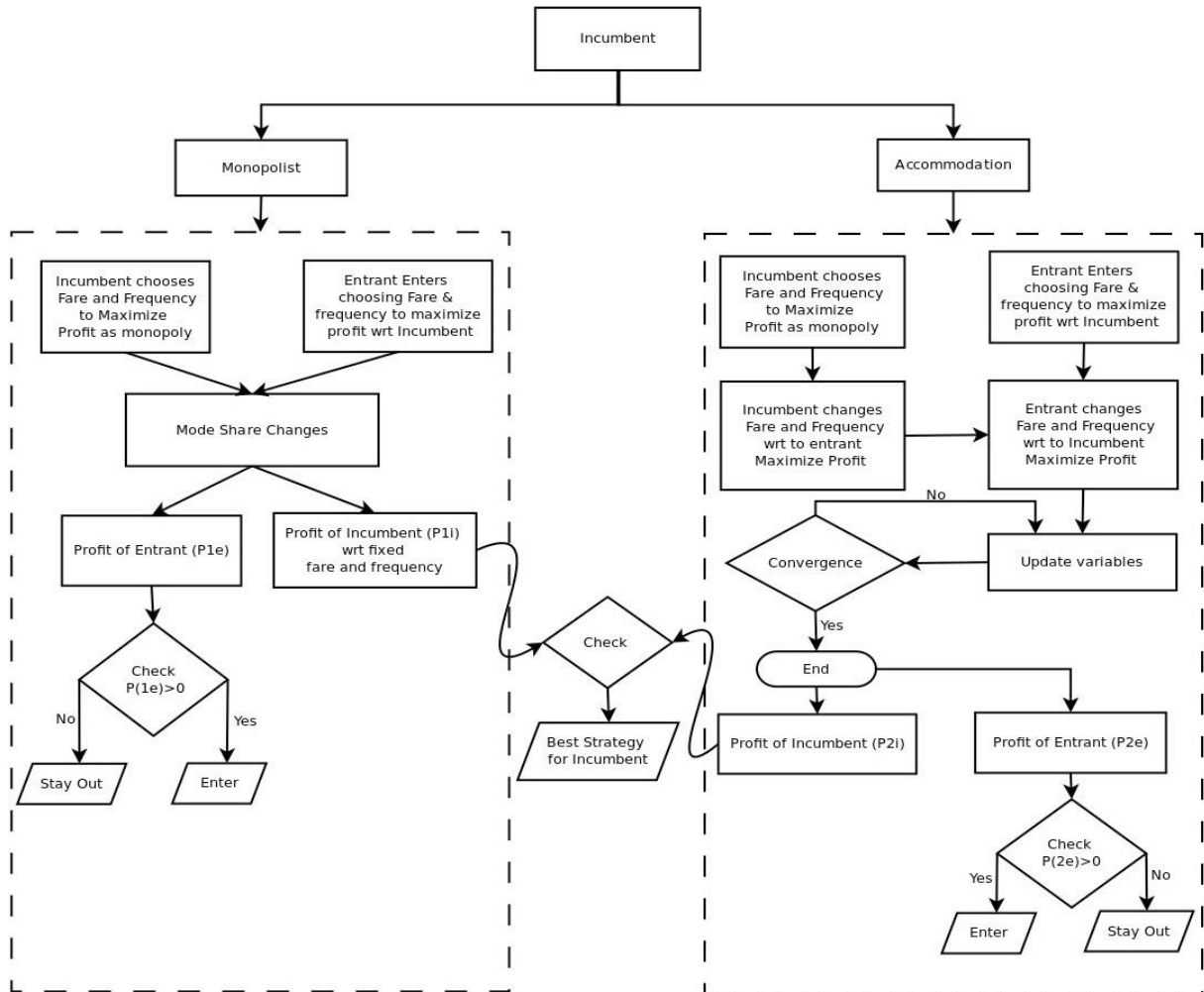


Figure 2: Flowchart describing strategy of entrant

the players and the impact of total demand and sunk cost is analyzed. Figure 2 shows the precise approach we use for the purpose of simulation.

The model developed is simulated using hypothetical data. The parameter values used in the simulation are given in the table 2.

The profit maximization problem as well as the constraint was a convex quadratic function and Lingo 14.0 was used for finding out the global maxima on a personal computer with 1.73 GHz CPU and 1028 M memory. First of all, the profit maximizing fare and frequency is calculated for bus at various levels of demand.

In monopoly case, it is observed that as the market demand increases, the frequency increases proportionally to cater to the increasing demand, while the fair remained constant thereby resulting in incremental profits for bus.

Table 2. Details of the hypothetical case in the numerical analysis

Category	Parameter	Value
Travel Time (in hours)	Travel time on train	3
	Travel time on bus	2.5
	Travel time on HSR	0.5
Travel cost by train	Fair associated with train	100
VOT Distribution (in Rs/Hr)	Upper bound of VOT	300
	Lower Bound of VOT	1
Bus operating cost	Fixed Cost (in Rs/Hr)	3000
	Variable Cost (in Rs/trip)	500
HSR operating cost	Fixed Cost (in Rs/Hr)	208000
	Variable Cost (in Rs/trip)	1500
Capacity	Bus	50
	HSR	720
Demand profile	Total demand (passenger/hr)	250-750
Sunk cost profile	Sunk cost	0-51493

After the bus sets its decision variables, HSR decides its own profit maximizing values for fare and frequency and eventually, the new profit for the bus is calculated which, for the given parameters, comes out to be lesser than the original profit without entry by HSR. Table 3 documents the results of the simulation.

Table 3. The accommodation profit maximization solution for various levels of sunk costs

Demand (passenger trips/hr)	Profit when sunk cost = 0		Profit when sunk cost = 16164		Profit when sunk cost = 33829		Profit when sunk cost = 51493	
	For bus	For HSR	For bus	For HSR	For bus	For HSR	For bus	For HSR
250.00	16164.33	11841.14	11841.14	0.00	11841.14	-17664.67	11841.14	-35328.67
500.00	33828.67	26682.27	26682.27	17664.00	26682.27	0.00	26682.27	-17664.33
750.00	51493.00	41523.41	41523.41	35329.00	41523.41	17664.00	41523.41	0.00

Next step is to iteratively solve for the market equilibrium that will be arrived at as a result of rationality and intelligence of the two players. Taking the monopolistic fare and frequency as the starting point, we will try to maximize the profit functions of bus and train iteratively while considering the revised values of decision variables of the other player simultaneously till the profits of individual players converge. While doing so, on one

hand, we make sure that the frequency of operation is kept limited to some reasonable value and on another, the order of fares of train, bus and HSR remains the same as desired by the VOT framework so that the whole formulation is theoretically viable. This analysis was done on various levels of demand and sunk cost values, thus providing interesting insights about the profitability of both the modes. It is seen that as the demand increases, the equilibrium profits and the decision variable values for both the modes increase in general. Also, it can be seen that increasing the levels of sunk costs has a potential to restrict the entry of the HSR by making its equilibrium profits negative. Figure 3 shows how the profit changes with changing levels of sunk cost. Tables 4 provides the accommodation fare and frequency variable values for various levels of total demand whereas table 5 gives a detailed picture of the profit obtained for different combinations of sunk cost and total demand by both the players.

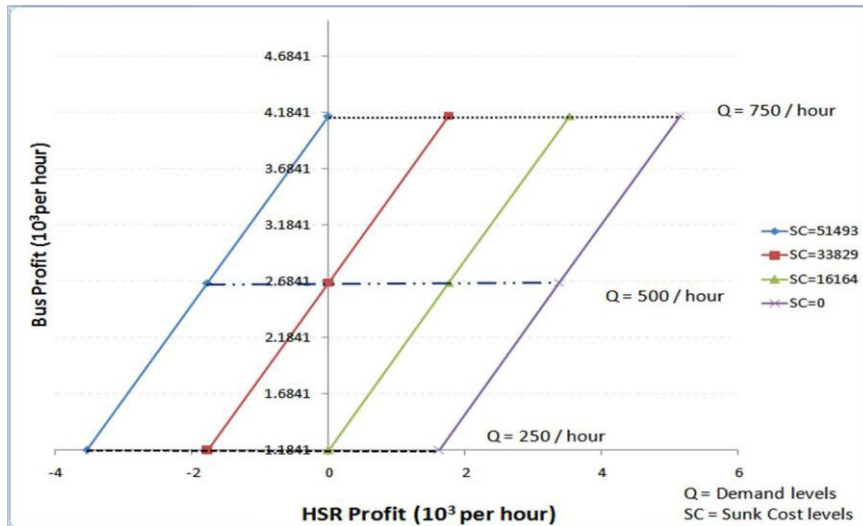


Figure 3: Profit comparisons for different sunk costs

Table 4. The accommodation profit maximization solution

Demand (passenger trips/hr)	For bus	For HSR	
	Frequency (trips/hr)	Fare (in Rs.)	Fare (in Rs.)
250	3.30	100	494.44
500	6.60	100	494.44
750	9.89	100	494.44

Table 5. The monopoly profit maximization solution

Demand (passenger trips/hr)	Frequency (trips/hr)	Fare (in Rs.)	Profit (before entry)	Profit (after entry)
250.00	4.01	130.00	21080.27	9093.31
500.00	8.02	130.00	45160.54	21186.62
750.00	12.04	130.00	69240.80	33279.94

6. Conclusions and Future Work

The paper formulates a game theoretic framework to analyze potential competition between the various modes of transport before and after HSR is introduced into the scenario and tries to derive some conclusions assisting in transportation policy decisions based on simulations performed for Bangalore Mysore corridor. The VOT concept is used while deriving the demand equilibrium and then profit maximization for various strategies is considered so that the sub game perfect Nash equilibrium could be found out using backward induction for various scenarios generated by varying sunk cost for HSR and total demand for the system.

The data based simulation analyzes the sensitivity of the equilibrium state to parameters. It is seen that when total demand in the market increases, players keep their fares to same levels and increase the frequency to cater to the changed demand levels. Thus, changing demand warrants recalculation of the equilibrium. Furthermore, demand is not the true criteria for determining whether HSR is feasible on a given corridor. The share of passengers having high VOT, meaning the share of passengers who are willing to pay more for the same amount of time, is the main factor. Thus the consideration of the distribution of VOT on every corridor is of supreme importance. One of the most important results is that the market entry of HSR will happen under accommodation strategy when demand is high and sunk cost is low. If the sunk cost is very high, even with high demand, HSR will never be able to obtain positive profits and staying out remains the only feasible strategy. At low demand and high sunk cost, HSR may undergo losses even if the bus plays a monopolist strategy. Thus, decision making bodies can look into options that reduce the infrastructure cost like upgrading the existing track, optimizing the operational costs, low power engines etc.

The proposed model can be improved further in various directions. The VOT approach used in developing the model currently uses uniform distribution but using log-normal distribution, which effectively takes income of the passengers into account, is a better approach and would lead to more meaningful results by allowing us to scrutinize sensitivity of varying the mean and the standard deviation of the income of the population on the resulting Nash equilibrium. Another way to analyze the demand equilibrium would be by considering other approaches such as discrete choice modeling which allow us to consider intangible preferences of the passengers.

Some more analysis about mergers and coalitions between various players is kept outside the purview of current problem, but will be a very interesting issue to analyze. Finally, it would be worthwhile to include more modes of transportation into competition and instead of looking at profit maximization for individual mode optimizing social welfare should be made an important consideration.

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